Econometric Inference Using Hausman Instruments

Jinyong Hahn — UCLA

Zhipeng Liao — UCLA

Nan Liu — Xiamen Univ.

Ruoyao Shi — UC Riverside

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Preview

- Idea underlying the Hausman IV is widely applied (IO, applied micro).
- We study its inference under a general asymptotic framework:
  - fixed n and $T \to \infty$;
  - $n \to \infty$ and fixed T;
  - $n \to \infty$ and $T \to \infty$.
- Stable convergence.
- We carefully study linear models:
  - No covariates – main idea;
  - With covariates – realistic applications.
- Nonlinear BLP models.
- Connections & differences to JIVE-type estimators.

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Benchmark Model: The Simplest Example in Empirical IO

- Demand equation of a product:

$$y_{i,t} = \alpha_i + \beta p_{i,t} + u_{i,t}, \ (i = 1, \ldots, n; \ t = 1, \ldots, T)$$

  - $y_{i,t}$ – **market share** of the product in location i at time t;
  - $p_{i,t}$ – **price** of the product in location i at time t;
  - α_i – **unobs.** demand shifters in location i.

- "First stage":

$$p_{i,t} = \eta_i + \gamma c_t + v_{i,t}, \ (i = 1, \ldots, n; \ t = 1, \ldots, T)$$

  - η_i – **unobs.** cost shifters in location i (e.g., transportation);
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Hausman IV & Estimator

- Idea: use price $p_{i',t}$ in **other** locations as proxy for c_t.

$$z_{i,t} \equiv \frac{1}{n-1} \sum_{i' \neq i} p_{i',t}.$$

- Hausman IV estimator:

$$\hat{\beta}_H \equiv \frac{\sum_{t \leq T} \sum_{i \leq n} (z_{i,t} - \bar{z}_{i,\cdot})(y_{i,t} - \bar{y}_{i,\cdot})}{\sum_{t \leq T} \sum_{i \leq n} (z_{i,t} - \bar{z}_{i,\cdot})(p_{i,t} - \bar{p}_{i,\cdot})} = \frac{\sum_{t \leq T} \sum_{i \leq n} z_{i,t} (y_{i,t} - \bar{y}_{i,\cdot})}{\sum_{t \leq T} \sum_{i \leq n} z_{i,t} (p_{i,t} - \bar{p}_{i,\cdot})}$$

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The Basic Theorem

Theorem 1 (Asymptotic distribution of $\hat{\beta}_H$)

Let $\sigma_c^2 \equiv plim_{nT\to\infty} T^{-1}\sum_{t=1}^{T}(c_t - \bar{c})^2$ and let $\mathcal{F}_0$ be the σ-field generated by $\{c_t\}_{t=1}^{T}$. Then under Assumption 1*, as $nT \to \infty$,*

$$\sqrt{nT}(\hat{\beta}_H - \beta) \longrightarrow \omega_\infty Z \quad (\mathcal{F}_\infty\text{-stably}),$$

where $Z \sim N(0,1)$, $\omega_\infty \perp Z$, and

$$\omega_\infty^2 \equiv \frac{\gamma^2\sigma_u^2\sigma_c^2 + (n-1)^{-1}(\sigma_u^2\sigma_v^2 + \sigma_{u,v}^2)}{\gamma^4\sigma_c^4}.$$

- $p_{i',t}$ enters both $z_{i,t}$ and $z_{i'',t}$ $\Rightarrow$ "clustering/interlinkage" even without clustering in the original model $\Rightarrow$ $\sigma_{u,v}^2$. More details
- This "clustering/interlinkage" matters if n is fixed.

The Basic Theorem (cont'd)

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- Theorem 1 allows for fixed n or fixed T, provided $nT \to \infty$.
- If T is fixed, then ω_∞^2 contains a random nuisance parameter σ_c^2.
- Stable convergence is weak convergence of conditional distribution given σ_c^2: stronger than usual "$\xrightarrow{d.}$", crucial for inference.

Classical Standard Errors

Lemma 1 (Classical standard error)

$$SE_0(\hat\beta_H) \equiv \sqrt{\frac{\left(\sum_{t\le T}\sum_{i\le n}(z_{i,t}-\bar z_{i,\cdot})^2\right)\left(\sum_{t\le T}\sum_{i\le n}\hat u_{i,t}^2\right)}{nT\left(\sum_{t\le T}\sum_{i\le n}(z_{i,t}-\bar z_{i,\cdot})p_{i,t}\right)^2}},$$

where $\hat u_{i,t} \equiv y_{i,t} - \bar y_{i,\cdot} - \hat\beta_H(p_{i,t}-\bar p_{i,\cdot})$. *Then under* Assumption 1,

$$\sqrt{nT}\,SE_0(\hat\beta_H) \xrightarrow{p.} \sqrt{\frac{\gamma^2\sigma_u^2\sigma_c^2 + (n-1)^{-1}(\sigma_u^2\sigma_v^2 + \cancel{\sigma_{u,v}^2})}{\gamma^4\sigma_c^4}}\cdot(1-T^{-1}).$$

- $\left(1-T^{-1}\right)^{-1/2}\cdot SE_0(\hat\beta_H)$ is a valid SE only if $n\to\infty$.

Clustered Standard Error

Lemma 2 (Clustered (at t) standard error)

$$SE_1(\hat{\beta}_H) \equiv \sqrt{\frac{\sum_{t\le T}\left(\sum_{i\le n}(z_{i,t}-\bar{z}_{i,\cdot})\hat{u}_{i,t}\right)^2}{\left(\sum_{t\le T}\sum_{i\le n}(z_{i,t}-\bar{z}_{i,\cdot})p_{i,t}\right)^2}}.$$

Then under Assumption 1*, only if $T\to\infty$,*

$$\sqrt{nT}SE_1(\hat{\beta}_H) \xrightarrow{p.} \sqrt{\frac{\gamma^2\sigma_u^2\sigma_c^2 + (n-1)^{-1}(\sigma_u^2\sigma_v^2+\sigma_{u,v}^2)}{\gamma^4\sigma_c^4}}.$$

Extended model

A Practical Proposal: Averaging Standard Error

- **Uniformly valid** SE (fixed n or fixed T OK, provided $nT \to \infty$):

$$SE_{avg}(\hat{\beta}_H) \equiv \frac{n}{T+n} \cdot \underbrace{\left(1 - T^{-1}\right)^{-1/2} \cdot SE_0(\hat{\beta}_H)}_{\text{valid only if } n \to \infty} + \frac{T}{T+n} \cdot \underbrace{SE_1(\hat{\beta}_H)}_{\text{valid only if } T \to \infty}$$

- Empirical applications cover:
  - fixed n and $T \to \infty$ (Hausman, 1996);
  - $n \to \infty$ and fixed T (Nevo, 2001);
  - and $n, T \to \infty$ (DellaVigna & Gentzkow, 2020).
- One simple unified formula.

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    - **fixed n and T** $\to \infty$ (Hausman, 1996);
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Extended model

Inference

Theorem 2

Under Assumption 1, as $nT \to \infty$, we have

$$\sqrt{nT}\, SE_{avg}(\hat{\beta}_H) \xrightarrow{p.} \omega_\infty,$$

and

$$\frac{\hat{\beta}_H - \beta}{SE_{avg}(\hat{\beta}_H)} \longrightarrow N(0,1) \quad (\mathcal{F}_\infty\text{-mixing}).$$

- This justifies statistical inference, such as

$$CI_{1-\alpha} = \left[\hat{\beta}_H - z_{\alpha/2} SE_{avg}(\hat{\beta}_H), \quad \hat{\beta}_H + z_{\alpha/2} SE_{avg}(\hat{\beta}_H)\right].$$

- Mixing convergence is stable convergence with a pivotal limit.
- Had it been usual "$\xrightarrow{d.}$" in Theorem 1, $(\hat{\beta}_H - \beta)/SE_{avg}(\hat{\beta}_H)$ may not converge to $N(0,1)$ if T is fixed (due to random $\hat{\sigma}_c^2$), even though $SE_{avg}(\hat{\beta}_H)$ is uniformly consistent.

An Alternative Interpretation of Benchmark Model

- Treatment effect of incarceration:

$$y_{i,t} = \beta p_{i,t} + u_{i,t}, \ (i = 1, \ldots, n; \ t = 1, \ldots, T)$$

  - $y_{i,t}$ – post-incarceration **outcome** of defendant i handled by judge t;
  - $p_{i,t}$ – **prison time** of defendant i handled by judge t;
  - (assuming $\alpha_i = 0$ for simplicity).

- "First stage":

$$p_{i,t} = \gamma c_t + v_{i,t}, \ (i = 1, \ldots, n; \ t = 1, \ldots, T)$$

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An Alternative Interpretation of Benchmark Model (cont'd)

- Idea: use $p_{i',t}$ of **other** defendants by the same judge as proxy for c_t.

$$z_{i,t} \equiv \frac{1}{n-1} \sum_{i' \neq i} p_{i',t}.$$

- The IV estimator in judge assignment design:

$$\begin{aligned} \hat{\beta}_{JIVE} &\equiv \frac{\sum_{t \leq T} \sum_{i \leq n} (z_{i,t} - \bar{z}_{i,\cdot})(y_{i,t} - \bar{y}_{i,\cdot})}{\sum_{t \leq T} \sum_{i \leq n} (z_{i,t} - \bar{z}_{i,\cdot})(p_{i,t} - \bar{p}_{i,\cdot})} \\ &= \frac{\sum_{t \leq T} \sum_{i \leq n} z_{i,t}(y_{i,t} - \bar{y}_{i,\cdot})}{\sum_{t \leq T} \sum_{i \leq n} z_{i,t}(p_{i,t} - \bar{p}_{i,\cdot})} \end{aligned}$$

An Alternative Interpretation of Benchmark Model (cont'd)

- Without covariates, **Hausman IV estimator** = **JIVE** in the judge assignment design with judge dummies as IVs (Angrist, Imbens and Krueger, 1999).
- Our uniform inference method covers empirical applications:
  - $\mathbf{n} \to \infty$ **and fixed T** (Dahl, Kostol and Mogstad, 2014; Aizer and Doyle Jr., 2015; Dobbie, Goldin and Yang, 2018; Autor et al., 2019; Norris, Pecenco and Weaver, 2021);
  - **fixed n and** $\mathbf{T} \to \infty$ (Doyle Jr., 2017; Gross and Baron, 2022);
  - $\mathbf{n}, \mathbf{T} \to \infty$ (Kling, 2006; Di Tella and Schargrodsky, 2013).
- (If $T \to \infty$ (many judges), a researcher **should** cluster at the t (judge) level, **in addition to** other levels informed by empirical context.)

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Simulations

Emprical Coverage Probabilities of the 0.95-Confidence Intervals

	$\rho = 0.5$			$\rho = 2.0$		
(n, T)	Classical	Cluster	Averaging	Classical	Cluster	Averaging
$(250, 2)$	0.843	0.000	0.945	0.824	0.000	0.919
$(250, 4)$	0.911	0.581	0.949	0.914	0.588	0.950
$(250, 8)$	0.934	0.781	0.949	0.933	0.779	0.947
$(2, 250)$	0.941	0.951	0.951	0.907	0.950	0.949
$(4, 250)$	0.944	0.946	0.946	0.929	0.948	0.948
$(8, 250)$	0.946	0.945	0.945	0.939	0.946	0.946
$(100, 100)$	0.949	0.937	0.945	0.947	0.936	0.944

Extended Model (With Covariates)

- Main equation:

$$y_{i,t} = \alpha_i + \beta p_{i,t} + \underline{w'_{i,t}\theta} + u_{i,t}, \ (i = 1, \ldots, n; \ t = 1, \ldots, T).$$

- Hausman IV:

$$z_{i,t} \equiv \frac{1}{n-1} \sum_{i' \neq i} p_{i',t}.$$

- Let $\tilde{y}_{i,t}$ and $\tilde{p}_{i,t}$ be $y_{i,t}$ and $p_{i,t}$ partialling out $w_{i,t}$, then the Hausman IV estimator:

$$\hat{\beta}_{H,ex} \equiv \frac{\sum_{t \leq T} \sum_{i \leq n} (z_{i,t} - \bar{z}_{i,\cdot})(\tilde{y}_{i,t} - \bar{\tilde{y}}_{i,\cdot})}{\sum_{t \leq T} \sum_{i \leq n} (z_{i,t} - \bar{z}_{i,\cdot})(\tilde{p}_{i,t} - \bar{\tilde{p}}_{i,\cdot})} = \frac{\sum_{t \leq T} \sum_{i \leq n} z_{i,t}(\tilde{y}_{i,t} - \bar{\tilde{y}}_{i,\cdot})}{\sum_{t \leq T} \sum_{i \leq n} z_{i,t}(\tilde{p}_{i,t} - \bar{\tilde{p}}_{i,\cdot})}$$

Extended Model (With Covariates, cont'd)

- Our basic theorem and averaging SE extend straightforwardly to this extended model, covering many IV scenario (i.e., $T \to \infty$).
- With covariates, the Hausman IV estimator differs from **IJIVE** (Ackerberg & Devereux, 2009), **UJIVE** (Kolesar, 2013), or **FEJIV** (Chao, Swanson & Woutersen, 2023), in how it handles the covariates.
- Judge assignment design features weak IVs, we defer to future work.
- PS: BLP Model with Hausman IV.

Thank you!

Assumptions for Theorem 1

Assumption 1

(i) $(u_{i,t}, v_{i,t})$ *are i.i.d. across* i *and* t *with* $\mathbb{E}[u_{i,t}] = 0$ *and* $\mathbb{E}[v_{i,t}] = 0$*;*
(ii) c_{t_1} *is independent of* (u_{i,t_2}, v_{i,t_2}) *for any* i*, and any* t_1 *and* t_2*;*
(iii) $\mathbb{E}[u_{i,t}^4] + \mathbb{E}[v_{i,t}^4] \leq K$ *and* $\max_t \mathbb{E}[c_t^4] \leq K$*;*
(iv) $\hat{\sigma}_c^2 \equiv T^{-1} \sum_{t=1}^{T} (c_t - \bar{c})^2 \to_p \sigma_c^2$ *where* $\sigma_c^2 > 0$ *almost surely;*
(v) $n \geq 2$*,* $T \geq 2$ *and* $(nT)^{-1} = o(1)$*.*

Details for Clustering/Interlinkage

The numerator of $\hat{\beta}_H - \beta$ decomposes to:

$$\sum_{t\leq T}\sum_{i\leq n} z_{i,t}\left(u_{i,t} - \bar{u}_{i,\cdot}\right) = \sum_{i\leq n}\sum_{t\leq T}\gamma u_{i,t}\left(c_t - \bar{c}\right) + \frac{1}{n-1}\sum_{t\leq T}\sum_{i\leq n}\sum_{i'\neq i} u_{i,t}v_{i',t} - \frac{T}{n-1}\sum_{i\leq n}\sum_{i'\neq i}\bar{u}_{i,\cdot}\bar{v}_{i',\cdot},$$

BLP Model with Hausman IV

1. In the BLP model (i denotes a market, t denotes a product)

$$\begin{aligned} y_{i,t} &= \tau_{i,t}(s_{i,1}, \ldots, s_{i,T}; \delta) \\ &= \alpha_i + \beta p_{i,t} + w_{i,t}'\theta + u_{i,t}, \; (i = 1, \ldots, n; \; t = 1, \ldots, T) \end{aligned}$$

   - $s_{i,t}$ – observed market share;
   - δ – parameters governing distribution of shocks & random coefficients.

2. GMM estimator chooses (β, θ, δ) to minimize the sample moments of

$$\mathbb{E}\left[\left(\tau_{i,t}(s_{1,1}, \ldots, s_{n,T}; \delta) - \alpha_i - \beta p_{i,t} - w_{i,t}'\theta\right) z_{i,t}\right] = 0,$$

   where the Hausman IV is still $z_{i,t} \equiv (n-1)^{-1} \sum_{i' \neq i} p_{i',t}$.

3. The "clustering/interlinkage" similar to the Extended model still happens in the "numerator" of the GMM estimator for fixed n.

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